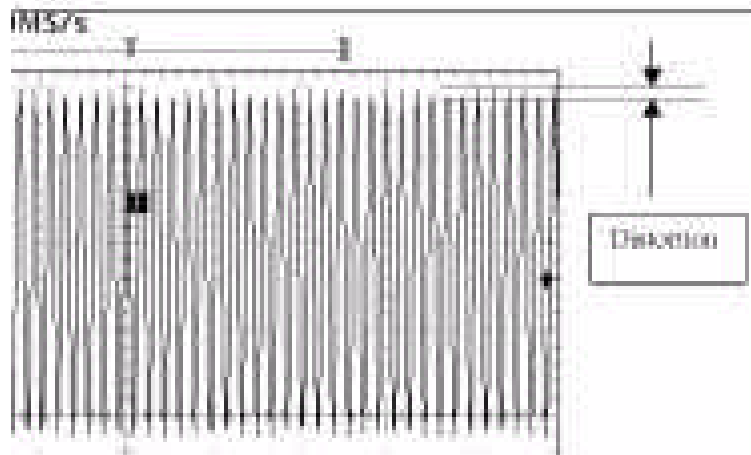


# Sub-Nyquist Distortions in Sampled One- and Two-Dimensional Signals Studied

Hardware testing frequently involves the acquisition of waveform and instrumentation signals, which are often recorded on waveform recorders, oscillographs, and video recorders. Years ago, the waveforms were viewed as analog records, as drawn by a paper strip chart pen or electron beam on a cathode ray tube screen. One of the problems in the past was that the analog electronics may not have been able to accurately display the full amplitude of a signal if the real-time signals exceeded the frequency response bandwidth of the recording device.

The advent of digital oscilloscopes, waveform recorders, and video frame-grabbers solved many of the frequency response problems, though not all. A restriction on digital waveform acquisition is well known by people in the instrumentation field. Put simply, the sampling frequency must be at least twice the frequency of any signal to be sampled, or vice versa; the signals must be filtered so that none of the signal frequencies are higher than one-half the sample rate (the Nyquist Limit). Then, per Shannon's Sampling Theorem (1949, ref. 1), any sampled signal can be reconstructed for viewing on a display device. If any signals exceed the Nyquist frequency limit, error signals called aliases occur in the output display.

The common assumption is that once the system signals and sample rates obey all the sampling requirements, there are no other front-end sampling issues. (There may be other requirements issues, such as having enough memory or disk space for storing all the samples.)



*Waveform peak distortion captured with a digital oscilloscope at 1 million samples per second. Input frequency was 6/53 of that, or 113.20754 kHz.*

Recently we showed (see preceding figure) that other sampling considerations apply when signals are much lower in frequency than the sample rate. Similar to a beat-note in an out-of-tune musical instrument, there is a distortion introduced into the displayed record that

is most noticeable at the signal waveform peaks. Investigations have shown that the distortion in the signal peaks results in a modulation similar to a form of amplitude modulation (AM) on a radio signal. The signal frequencies of concern are those that are a fraction of a harmonic of the sample rate, in which the numerical ratio in the fraction has all the common primes factored out.

Unfortunately, there are literally hundreds of possible ratios (such as 6/53 in the figure) that can cause the distortion. The impact of having these false modulation signals in the record depends on the application. Through no fault of the manufacturers of waveform recorders and oscilloscopes, these distortion products can confuse and frustrate customers who are not familiar with the implications of sampling. A full publication on this topic is available (ref. 2).

## References

1. Shannon, C.E.: Communication in the Presence of Noise. Proc. IRE, vol. 37, Jan. 1949, pp. 10-21.
2. Williams, G.L.: Sub-Nyquist Distortions in Sampled Data, Waveform Recording, and Video Imaging. NASA/TM--2000-210381, 2000. <http://gltrs.grc.nasa.gov>.

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